LIE GROUP SHAPE MODELS: A CORE TECHNOLOGY FOR A COMPUTER-ASSISTED SURGERY OPEN FRAMEWORK

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INTRODUCTION

Conventional computer-assisted orthopaedic surgery (CAOS) involves many time-consuming and patient-dependent repeatable tasks in which automation is highly desirable. Relevant work includes the development of efficient and accurate automatic solutions for CAOS tasks such as region-of-interest segmentation (Chu 2014), implant fitting (Wu 2015), and surgical planning (Hefny 2015). Automation requires mathematical descriptors of the human anatomy. Polynomial descriptors are infeasible due to the high variability of the anatomy; statistical models, derived from human populations, are promising alternatives.

We propose an open framework and a core technology for CAOS tasks. An open software framework enables users to easily exchange modules to customize applications. Here, the framework consists of one core and multiple application modules. The core is a Lie group statistical shape model constructed from a representative population. The core constructs a shape model, or atlas, for each region of interest. The atlas is constructed once, after which it can be used by different application modules such as region-of-interest (ROI) segmentation, implant fitting, or surgical planning. Figure 1 shows the component structure of the framework.

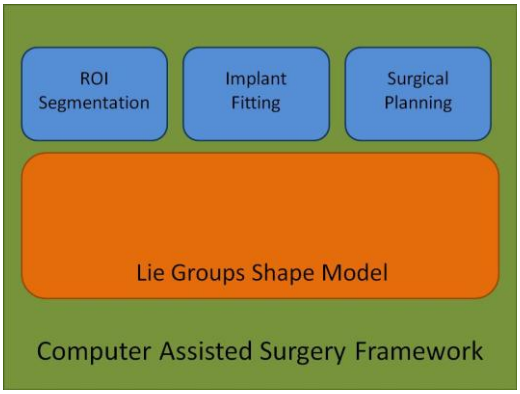

Figure 1: A computer-assisted orthopaedic surgery open framework with a Lie group shape model core. The orange component is the core of the framework and the blue components are the application modules.

A statistical shape model is a mathematical structure representing the variability of a certain shape, using measures of statistics that come from geometrical descriptions of samples of a population. A highly influential development was the point distribution model (Cootes 1995), which has been widely used in many general applications. A point distribution model relies on principal component analysis and assumes that shapes can be described in a Euclidean space. Shapes that are better described in a non-linear space confound such analytical methods.

Matrix Lie groups representations of shapes are a new promising way of discrete shape representation that allows processing data in non-linear spaces while performing simple computations (Hefny 2013a, 2013b, 2014a, 2014b). The power of a Lie group is that its structures can be simultaneously manipulated using algebra and geometry.

We describe the construction of the Lie group statistical shape model in the methods section. Then, we compare the performance of the method to the established linear principal component analysis. Once a model is constructed for a particular anatomy, it can be used with multiple applications.

METHODS

Model construction requires a data set of a population of triangular meshes representing surfaces of anatomical samples. A one-to-one triangular correspondence must be established between surfaces. Each surface can be processed as an ensemble of triangles. For the rest of this section, the method will be described for one triangle but the discussion can be effortlessly extended to the ensemble, or equivalently to the full surface.

For a set of triangles $\{A_1,A_2,...,A_n\}$ in 3D Cartesian space, a base triangle B is approximated as an intrinsic mean of shapes on the manifold (Pennec 2006). This is analogous to the computation of mean on Euclidean space in linear statistics.

Each triangle A_i can be translated, by a distance d, and rotated, by a rotation matrix R, to its canonical form using a unique homogeneous transformation T. The transformation maps one of the triangle vertices to the origin, one of the edges to the Cartesian *x-axis*, and maps the triangle plane to the Cartesian *xy-plane*. A similar transformation can be applied to the mean triangle B. By setting all values of the *z-dimension* to zero, this transformation maps the 3D triangle to a 2D one. Each canonical triangle A_i can be scaled, by a scaling matrix S, and sheared, by a shearing matrix H, to the canonical triangle B using a unique deformation matrix $D=SH$. As a result of the uniqueness of the transformation and deformation matrices, any triangle A_i can be transformed to the triangle B and retrieved back. The homogeneous transformation matrices are elements of a Lie group (Hefny 2014a), and the 2D deformation matrices are elements of a Lie group (Hefny 2013a).

A new Lie group can be constructed as the product of two parent Lie groups. A particularly convenient representation of the product of two matrix Lie groups is to create a larger matrix with a block-diagonal structure. The transformation and deformation matrices do not interact through the new group operation, which implies that the properties of the parent groups are preserved. The great utility of the representation is that it decouples the rigid transformation of a triangle in 3D space from its canonical shape. This has 9 degrees of freedom, which is a lower dimensional representation than using a 3D affine transformation that has 12 degrees of freedom. Figure 2, illustrates triangles A_i and B and their canonical forms.

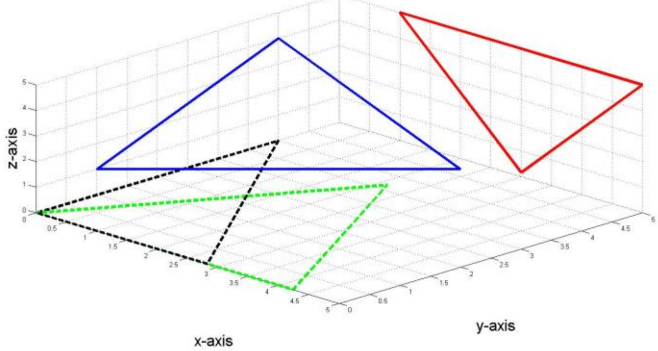

Figure 2: Triangle *A* (blue) is transformed to the canonical form (green), deformed to the canonical for of triangle *B* (black), and transformed to the original triangle *B* (red).

The logarithm of the matrix is an equivalent vector on the tangent space of the matrix manifold at the identity matrix. The matrix exponential is the reverse mapping from the tangent to the matrix manifold. As a result, matrices can be mapped to the tangent space for linear statistical analysis and mapped back to the original manifold. For model construction, singular value decomposition, a robust version of principal component analysis, is applied to the shapes on the tangent space.

RESULTS

A dataset of 15 proximal femur surfaces were extracted from Computed Tomography (CT) scans of patients who underwent computer-assisted hip resurfacing surgery. The data collection was approved by the relevant Institutional Review Board (IRB). The images were acquired using a GE Lightspeed 16-slice CT scanner (General Electric, Milwaukee, USA) with a moving gantry; axial images had a 1.25mm slice thickness reconstructed from automatic tube-current modulation. The dataset was processed to establish triangular correspondence between samples. Both PCA and Lie group analysis were applied to the proximal femur dataset, from which selected femurs were reconstructed using the accumulation of components.

Table 1 shows the contribution percentages of principal components for both linear PCA and non-linear Lie group analysis. For PCA, the first component covered around 37% of the data variation and the second component covered around 27%. Each of the first 9 components had a contribution greater than 1%. For Lie group analysis, the first component covered around 83% of the data variation and the second component covered around 8%. Each of the first 4 components had a contribution greater than 1%. These results indicate that the first component of the Lie group analysis captured most of the variation of the dataset whereas at least 5 components were required to capture the equivalent percentage for PCA.

Principal Component	**PCA (%)**	**Lie group (%)**
1	37.33	82.62
2	27.70	8.08
3	11.66	4.4
4	10.46	2.48
5	4.30	0.96
6	2.81	0.39
7	2.00	0.33
8	1.24	0.24
9	1.05	0.16
10	0.78	0.11

Table 1: This table shows the percentage of contribution of the first 10 principal components for both linear PCA and non-linear Lie group analysis.

Figures 3 and 4, show the plotted singular values and accumulated sums for the linear PCA and the non-linear Lie group analysis, respectively. The plots show that only 3 components were required to cover 95% from the population, whereas 7 components were required to cover the same percentage in PCA.

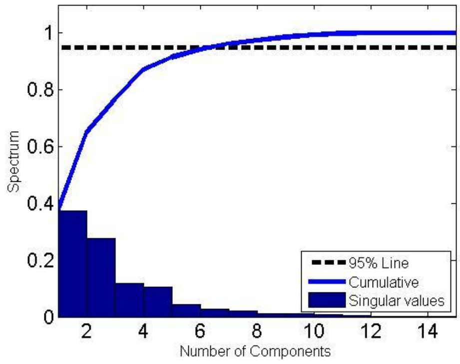

Figure 3: Singular values and accumulated sums for principal component analysis.

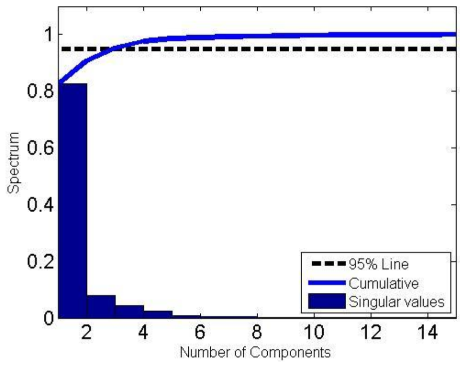

Figure 4: Singular values and accumulated sums for Lie group analysis.

DISCUSSION

This paper presented an open framework for the automation of CAOS repeatable tasks with a robust Lie group statistical shape model core. The open framework enables development of surgical application modules without changing the core. Also, it easily allows future extensions and enhancements of applications. The core of the module is a novel product Lie group statistical shape model. This model is based on triangular meshes, which are the most common 3D shape representation of surfaces extracted from medical images. The use of Lie groups enables simple implementation of manifold statistical analysis that may outperform linear analysis, particularly in medical imaging.

The method modelled 3D surfaces as an ensemble of triangles, or indeed any three non-collinear points. The method transformed each base triangle to its canonical equivalent, deformed it to the canonical equivalent of the corresponding triangle, and transformed it to the altered triangle. Both the transformation matrix and the deformation matrix are Lie groups, hence the product group is a Lie group. This enabled simple Lie group computations because an element in a matrix Lie group is also an operator on the manifold: the computations corresponded to an exact geodesic in the nonlinear manifold.

The method was validated using a dataset of 15 human proximal femurs. The proximal femur is a highly variable bony anatomy. The study showed that is such a small dataset, the Lie group based model outperformed the linear model in terms of compactness.

Linear PCA failed to capture the variations of the dataset, requiring 7 components to capture 95% of the variation within the simple dataset. The first principal component covered around 37% and the second component contributed around 28% of the population.

The product Lie group analysis was able to capture 95% of the variation with only 3 components. The first component alone captured around 83% of the dataset.

This paper has two conceptual contributions to the automation of CAOS. First, the open framework concept facilitates the development of automatic applications by utilizing an existing core for each anatomical region of interest. The statistical-model core suggests the applicability of the framework to a wide range of patients from the same population. The framework can be adapted to new applications, other than the ones mentioned earlier, without restriction.

Second, using the product Lie group representation for non-linear statistical analysis is an efficient and accurate processing core technology for the framework. The computations involved in the construction of the shape model are simple and accurate.

In conclusion, the open framework with a Lie group statistical shape model core promises to enhance the progress of the automation of CAOS repeatable tasks in a accurate and efficient manner. Consequently, this can enhance the outcomes of CAOS procedures.

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